

# Computation with Absolutely No Space Overhead

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Technical University of Berlin

Developments in Language Theory Conference, 2003



# Outline

- 1 The Model of Overhead-Free Computation
  - The Standard Model of Linear Space
  - Our Model of Absolutely No Space Overhead
- 2 The Power of Overhead-Free Computation
  - Palindromes
  - Linear Languages
  - Context-Free Languages with a Forbidden Subword
  - Languages Complete for Polynomial Space
- 3 Limitations of Overhead-Free Computation
  - Linear Space is Strictly More Powerful



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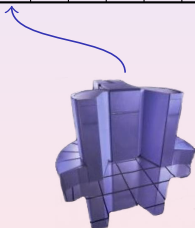
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# The Standard Model of Linear Space

tape

0	0	1	0	0	1	0	0
---	---	---	---	---	---	---	---



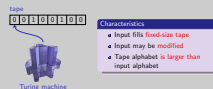
Turing machine

## Characteristics

- Input fills **fixed-size** tape
- Input may be **modified**
- Tape alphabet **is larger than** input alphabet

## Computation with Absolutely No Space Overhead

- └ The Model of Overhead-Free Computation
  - └ The Standard Model of Linear Space
    - └ The Standard Model of Linear Space



1. Point out that \$ is a marker symbol.
2. Stress the larger tape alphabet.

# The Standard Model of Linear Space

tape

\$	0	1	0	0	1	0	0
----	---	---	---	---	---	---	---



Turing machine

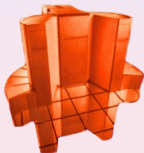
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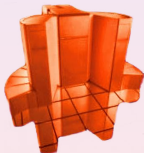
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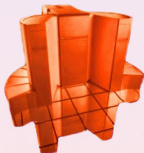
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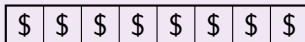
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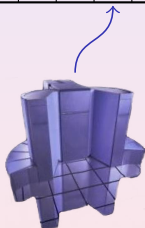
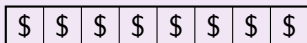
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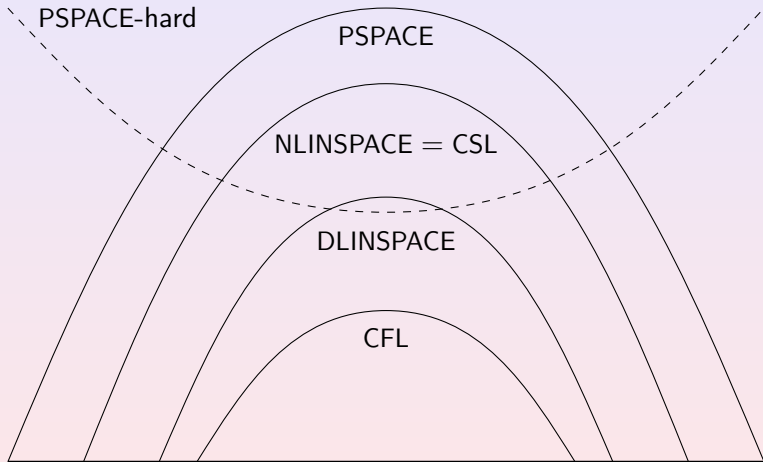


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# Linear Space is a Powerful Model

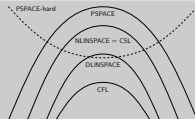


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## Computation with Absolutely No Space Overhead

- └ The Model of Overhead-Free Computation
  - └ The Standard Model of Linear Space
    - └ Linear Space is a Powerful Model

Linear Space is a Powerful Model



1. Explain CSL.
2. Point out the connections to formal language theory.

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# Our Model of “Absolutely No Space Overhead”

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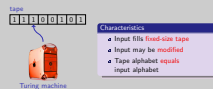
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## Our Model of “Absolutely No Space Overhead”



1. Point out that no markers are used.

# Our Model of “Absolutely No Space Overhead”



Turing machine

## Intuition

- Tape is used like a RAM module.

# Definition of Overhead-Free Computations

## Definition

A Turing machine is **overhead-free** if

- 1 it has only a single tape,
- 2 writes only on input cells,
- 3 writes only symbols drawn from the input alphabet.

# Overhead-Free Computation Complexity Classes

## Definition

A language  $L \subseteq \Sigma^*$  is in

**DOF** if  $L$  is accepted by a deterministic overhead-free machine with input alphabet  $\Sigma$ ,

$\text{DOF}_{\text{poly}}$  if  $L$  is accepted by a deterministic overhead-free machine with input alphabet  $\Sigma$  in polynomial time.

$\text{NOF}$  is the nondeterministic version of  $\text{DOF}$ ,

$\text{NOF}_{\text{poly}}$  is the nondeterministic version of  $\text{DOF}_{\text{poly}}$ .



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## Computation with Absolutely No Space Overhead

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- NDOF** is the nondeterministic version of DOF.

*(DOF is the deterministic version of DOF.)*

### 1. Joke about German pronunciation

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- NDOF** is the nondeterministic version of DOF.

NDOF<sub>poly</sub> is the nondeterministic version of DOF<sub>poly</sub>.

## 1. Stress meaning of D and N.

# Overhead-Free Computation Complexity Classes

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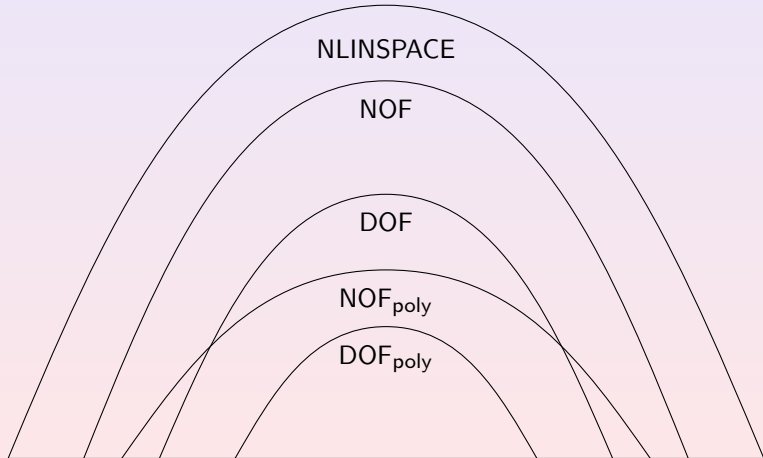
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# Simple Relationships among Overhead-Free Computation Classes



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# Palindromes Can be Accepted in an Overhead-Free Way

tape

0	0	1	0	0	1	0	0
---	---	---	---	---	---	---	---



overhead-free machine

## Algorithm

Phase 1:

Compare first and last bit

Place left end marker

Place right end marker

Phase 2:

Compare bits next to end markers

Find left end marker

Advance left end marker

Find right end marker

Advance right end marker

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# Computation with Absolutely No Space Overhead

## └ The Power of Overhead-Free Computation

### └ Palindromes

#### └ Palindromes Can be Accepted in an Overhead-Free Way

Palindromes Can be Accepted in an Overhead-Free Way



#### Algorithm

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Compare first and last bit  
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Place right end marker

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Advance right end marker

Use 3 minutes.

# Palindromes Can be Accepted in an Overhead-Free Way

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**Find right end marker**

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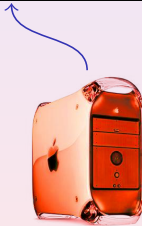
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Advance right end marker

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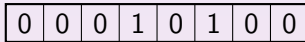
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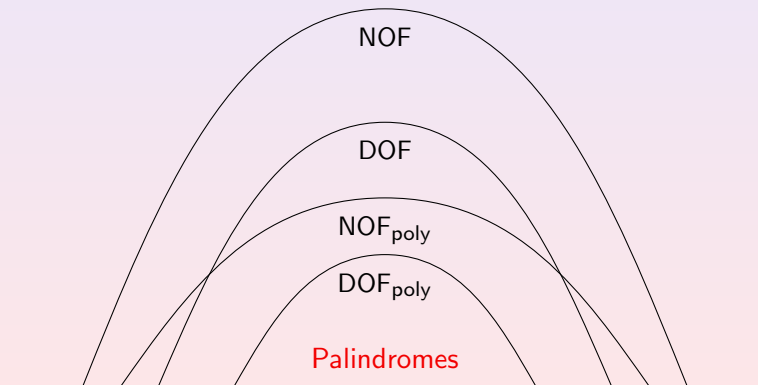
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# A Review of Linear Grammars

## Definition

A grammar is **linear** if it is context-free and there is only one nonterminal per right-hand side.

## Example

$G_1: S \rightarrow 00S0 \mid 1$  and  $G_2: S \rightarrow 0S10 \mid 0$ .

## Definition

A grammar is **deterministic** if  
“there is always only one rule that can be applied.”

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$G_1: S \rightarrow 00S0 \mid 1$  is deterministic.  
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##### Example

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 $G_2: S \rightarrow 0S10 \mid 0$  is **not** deterministic.

Just explain intuition.

# Deterministic Linear Languages Can Be Accepted in an Overhead-Free Way

## Theorem

*Every deterministic linear language is in  $\text{DOF}_{\text{poly}}$ .*



# Metalinear Languages Can Be Accepted in an Overhead-Free Way

## Definition

A language is **metalinear** if it is the concatenation of linear languages.

## Example

TRIPLE-PALINDROME =  $\{uvw \mid u, v, \text{ and } w \text{ are palindromes}\}$ .

## Theorem

*Every metalinear language is in  $\text{NOF}_{\text{poly}}$ .*



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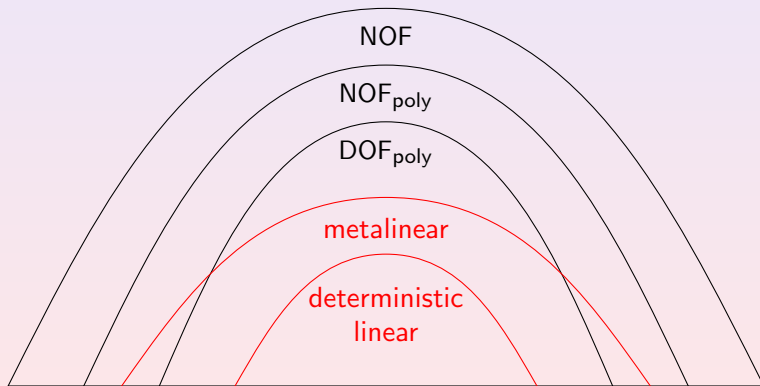
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## Theorem

*Every metalinear language is in  $\text{NOF}_{\text{poly}}$ .*

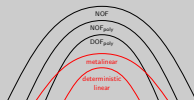


# Relationships among Overhead-Free Computation Classes



## Computation with Absolutely No Space Overhead

- └ The Power of Overhead-Free Computation
  - └ Linear Languages
    - └ Relationships among Overhead-Free Computation Classes



1. Skip next subsection if more than 18 minutes have passed.

# Outline

- 1 The Model of Overhead-Free Computation
  - The Standard Model of Linear Space
  - Our Model of Absolutely No Space Overhead
- 2 The Power of Overhead-Free Computation
  - Palindromes
  - Linear Languages
  - Context-Free Languages with a Forbidden Subword
  - Languages Complete for Polynomial Space
- 3 Limitations of Overhead-Free Computation
  - Linear Space is Strictly More Powerful



# Definition of Almost-Overhead-Free Computations

## Definition

A Turing machine is **almost-overhead-free** if

- 1 it has only a single tape,
- 2 writes only on input cells,
- 3 writes only symbols drawn from the input alphabet plus one special symbol.

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# Context-Free Languages with a Forbidden Subword Can Be Accepted in an Overhead-Free Way

## Theorem

*Let  $L$  be a context-free language with a forbidden word.  
Then  $L \in \text{NOF}_{\text{poly}}$ .*

» Skip proof



# Context-Free Languages with a Forbidden Subword Can Be Accepted in an Overhead-Free Way

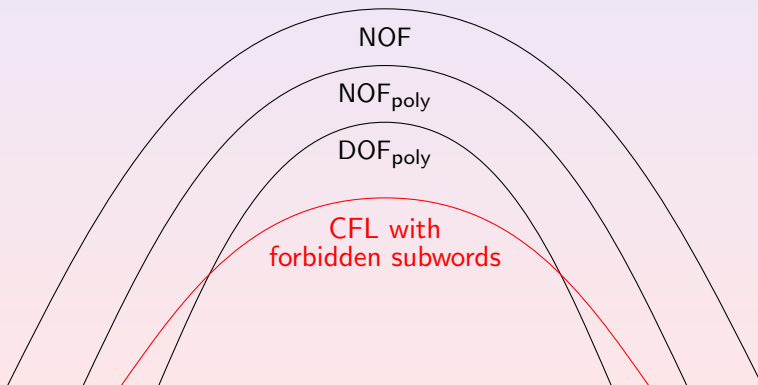
## Theorem

*Let  $L$  be a context-free language with a forbidden word.  
Then  $L \in \text{NOF}_{\text{poly}}$ .*

## Proof.

Every context-free language can be accepted by a nondeterministic almost-overhead-free machine in polynomial time.  $\square$

# Relationships among Overhead-Free Computation Classes



# Outline

- 1 The Model of Overhead-Free Computation
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# Overhead-Free Languages can be PSPACE-Complete

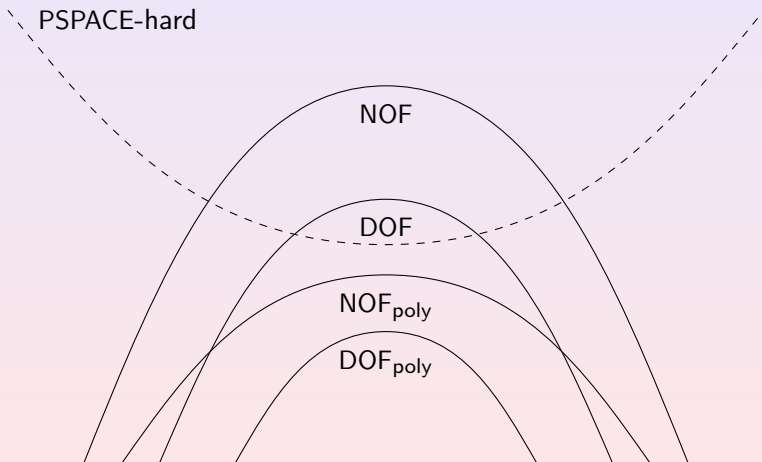
## Theorem

DOF *contains languages that are complete for PSPACE.*

► Proof details



# Relationships among Overhead-Free Computation Classes



# Outline

- 1 The Model of Overhead-Free Computation
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# Some Context-Sensitive Languages Cannot be Accepted in an Overhead-Free Way

## Theorem

$\text{DOF} \subsetneq \text{DLINSPACE}$ .

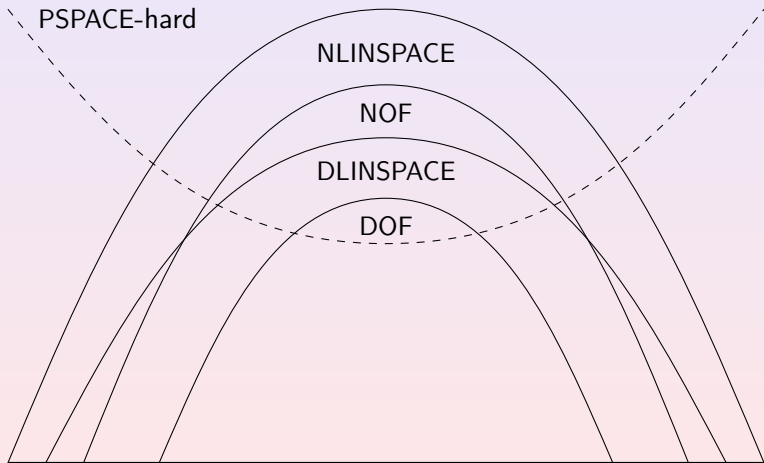
## Theorem

$\text{NOF} \subsetneq \text{NLINSPACE}$ .

The proofs are based on old diagonalisations due to Feldman, Owings, and Seiferas.



# Relationships among Overhead-Free Computation Classes



# Candidates for Languages that Cannot be Accepted in an Overhead-Free Way

## Conjecture

DOUBLE-PALINDROMES  $\notin$  DOF.

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$\{ww \mid w \in \{0,1\}^*\} \notin$  NOF.

Proving the first conjecture would show  $\text{DOF} \subsetneq \text{NOF}$ .



# Candidates for Languages that Cannot be Accepted in an Overhead-Free Way

## Theorem

$\text{DOUBLE-PALINDROMES} \in \text{DOF}.$

## Conjecture

$\{ww \mid w \in \{0,1\}^*\} \notin \text{NOF}.$

Proving the first conjecture would show  $\text{DOF} \subsetneq \text{NOF}.$



# Summary

- Overhead-free computation is a more faithful **model of fixed-size memory**.
- Overhead-free computation is **less powerful** than linear space.
- **Many** context-free languages can be accepted by overhead-free machines.
- We conjecture that **all** context-free languages are in  $\text{NOF}_{\text{poly}}$ .
- Our results can be seen as new results on the power of **linear bounded automata with fixed alphabet** size.



## Computation with Absolutely No Space Overhead

Summary

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- Overhead-free computation is a more faithful *model of fixed-size memory*.
- Overhead-free computation is *less powerful* than linear space.
- *Many* context-free languages can be accepted by overhead-free machines.
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- Our results can be seen as new results on the power of *linear bounded automata with fixed alphabet size*.

1. Point out result concerning all context-free languages.
2. Relationship to restart automata.

# For Further Reading



A. Salomaa.

*Formal Languages.*

Academic Press, 1973.



E. Dijkstra.

Smoothsort, an alternative for sorting in situ.

*Science of Computer Programming*, 1(3):223–233, 1982.



E. Feldman and J. Owings, Jr.

A class of universal linear bounded automata.

*Information Sciences*, 6:187–190, 1973.



P. Jančar, F. Mráz, M. Plátek, and J. Vogel.

Restarting automata.

*FCT Conference 1995*, LNCS 985, pages 282–292. 1995.



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# Appendix Outline

- 4 Appendix
  - Complete Languages
  - Improvements for Context-Free Languages
  - Abbreviations



# Overhead-Free Languages can be PSPACE-Complete

## Theorem

*DOF contains languages that are complete for PSPACE.*

## Proof.

- ❶ Let  $A \in \text{DLINSPACE}$  be PSPACE-complete.  
Such languages are known to exist.
- ❷ Let  $M$  be a linear space machine that accepts  $A \subseteq \{0, 1\}^*$   
with tape alphabet  $\Gamma$ .
- ❸ Let  $h: \Gamma \rightarrow \{0, 1\}^*$  be an isometric, injective homomorphism.
- ❹ Then  $h(L)$  is in DOF and it is PSPACE-complete. □

◀ Return



# Improvements

## Theorem

- 1  $\text{DCFL} \subseteq \text{DOF}_{\text{poly}}$ .
- 2  $\text{CFL} \subseteq \text{NOF}_{\text{poly}}$ .



# Explanation of Different Abbreviations

DOF	Deterministic Overhead-Free.
NOF	Nondeterministic Overhead-Free.
DOF <sub>poly</sub>	Deterministic Overhead-Free, polynomial time.
NOF <sub>poly</sub>	Nondeterministic Overhead-Free, polynomial time.

Table: Explanation of what different abbreviations mean.

