Supplementary PROJ.4 Notes— Swiss Oblique Mercator Projection

August 5, 1995
Release 4.3.3
by Gerald I. Evenden

This projection is an oblique Mercator that is based upon a method involving three steps:

1. conformally transform ellipsoid coordinates to a sphere,
2. transform the spherical system so that the specified projection origin will lie on the equator, and
3. perform spherical Mercator conversion to cartesian coordinates.

The projection name used here is tentative and is derived from it use with the Swiss coordinate system CH1903.

This projection is selected in the PROJ.4 system by `+proj=somerc` with parameters `+lon_0=` and `+lat_0` required to specify projection origin. Optionally, false easting and northing, `+x_0=` and `+y_0`, may be used as well as origin scale factor `k_0`. For usage with the Swiss CH1903 system, `+init=world:CH1903` may be used.

Acknowledgement

The development presented here was due to the kind assistance of Daniel Ebneter of the University of Bern who contributed invaluable documentation, code and other information that allowed derivation of the basic projection and provided benchmark values.

Forward Projection

The first step is the conversion of geodetic latitude to a sphere by the following equation:

$$\begin{aligned} \ln\tan\left(\frac{\pi}{4}+\frac{\phi'}{2}\right) &= c\left[\ln\tan\left(\frac{\pi}{4}+\frac{\phi}{2}\right)-\frac{e}{2}\ln\left(\frac{1+e\sin\phi}{1-e\sin\phi}\right)\right]+K && (1)\\ \lambda' &= c(\lambda-\lambda_0) && (2)\end{aligned}$$

where ϕ is latitude on the ellipsoid, ϕ' is latitude on the sphere and e is ellipsoid eccentricity. The factors c and K are determined from the designated latitude origin of the projection: ϕ_0. First, c is obtained from:

$$c=\left(1+\frac{e^2\cos^4\phi_0}{1-e^2}\right)^{1/2}$$

Next the equivalent origin latitude on the sphere is obtained by:

$$\sin\phi_0'=\frac{\sin\phi_0}{c}$$

Substitute ϕ_0 and ϕ_0' into Eqn. 1 and solve for K.

Next the spherical coordinates, ϕ' and λ', are rotated by ϕ_0 about an axis through the equator and perpendicular to the plane of the central meridian (Wray's *simple oblique*) so that the latitude ϕ_0 becomes $\phi''=0$ at λ_0:

$$\begin{aligned} \sin\phi'' &= \cos\phi_0'\sin\phi'-\sin\phi_0'\cos\phi'\cos\lambda' && (3)\\ \sin\lambda'' &= \cos\phi'\sin\lambda'/\cos\phi'' && (4)\end{aligned}$$

Finally, transform the rotated coordinates to cartesian by the spherical form of the Mercator projection:

$$y = R \ln \tan\left(\frac{\pi}{4} + \frac{\phi''}{2}\right) + y_0 \tag{5}$$
$$x = R\lambda'' + x_0 \tag{6}$$

where R is the geometric mean of the merdinal and parallel radius at the reference point:

$$R = k_0 a \frac{\sqrt{1-e^2}}{1 - e^2 \sin^2 \phi_0}$$

where a is the major axis of the ellipsoid. The scaling factor, k_0, is not used $(=1)$ in the Swiss version but it can reduce overall scale error if appropriately applied.

Inverse Projection

To get the cartesian coordinates back into the rotated spherical system:

$$\phi'' = 2\left[tan^{-1} \exp\left(\frac{y - y_0}{R}\right) - \frac{\pi}{4}\right] \tag{7}$$
$$\lambda'' = \frac{y - y_0}{R} \tag{8}$$

Rotate the spherical coordinates back to the original position:

$$\sin\phi' = \cos\phi'_0 \sin\phi'' + \sin\phi'_0 \cos\phi'' \cos\lambda'' \tag{9}$$
$$\sin\lambda' = \cos\phi'' \sin\lambda'' / \cos\phi' \tag{10}$$

The ellipsoid longitude is obtained from Eqn. 2 but latitude requires application of Newton-Raphson method for a solution of Eqn. 1: $x_{n+1} = x_n - f(x_n)/f'(x_n)$. The correction ratio is:

$$\left[C + \ln\tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right) - \frac{e}{2}\ln\left(\frac{1 + e\sin\phi}{1 - e\sin\phi}\right)\right] \frac{1 - e^2\sin^2\phi}{1 - e^2}$$

where:

$$C = \frac{1}{c}\left[K - \ln\tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\right]$$

Use ϕ' as the initial estimate of ϕ and iterate until the correction ratio has reached sufficient tolerance. The function is rapidly convergent.

Standard Definition

The following entry has been placed in the `world` file to facilitate CH1903 usage with the PROJ.4 system by specifying `+init=world:CH1903`:

```
<CH1903> # Swiss Coordinate System
    +proj=somerc +lat_0=46d57'8.660"N +lon_0=7d26'22.500"E
    +ellps=bessel +x_0=600000 +y_0=200000
    +k_0=1.  no_defs <>
```

Note, the original are "reverse engineered" from decimal values as I suspect original specifications were in DMS format—needs verification.